TSIN02 Internetworking

Exercise class 4 problems

Exercise 1: An ISP performs a network upgrade to meet the ever increasing data demand. It decides that the customers will pay $k = 150$ SEK/GB. The customers' price-demand curve is shown in Fig.1. We are using that the underlying utility function is based on a user poll where the users tell how much they are willing to pay for different amounts of data; that the utility function is concave; that the derivative of the utility function is invertible; and that the utility function evaluated for 0 GB is 0.

a) What is the net utility measured in SEK for a customer under usage-based pricing?
b) What is the net utility measured in SEK for a customer under flat rate?
c) Interpret the results! Especially comment on the difference in net utility in (a) and (b)!

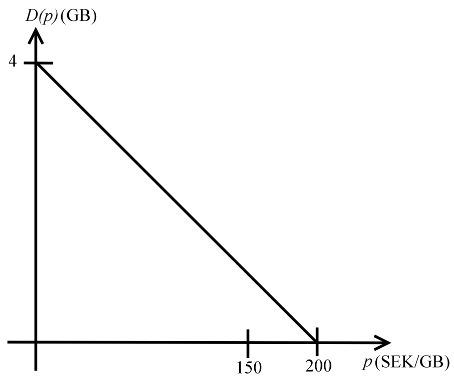

Figure 1: Demand $D(p)$ as a function of price p in Exercise 1.

Exercise 2: An ISP performs a network upgrade to meet the ever increasing data demand. The customers' price-demand curves before and after the upgrade are shown in Fig.2 and Fig.3. We are using that the underlying utility functions are based on user polls where the users tell how much they are willing to pay for different amounts of data; that the utility functions are concave; that the derivative of the utility functions are invertible; and that the utility functions evaluated for 0 GB are 0. The demand curves are shown in Fig. 2 and Fig.3.

The customers pay 50 SEK/GB flat rate before the service upgrade. Someone within the ISP suggests that the users should pay 300 SEK/GB flat rate after the upgrade to cover upgrade costs. Is it good or bad idea to upgrade with this new pricing scheme? Motivate your answer by careful calculation!

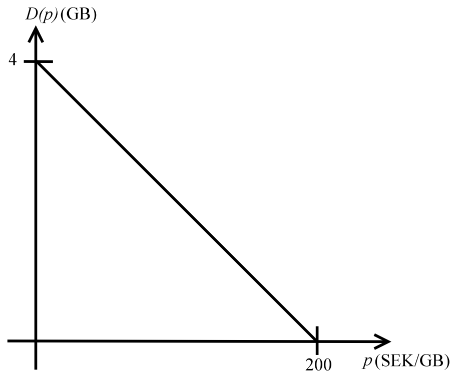

Figure 2: Demand $D(p)$ as a function of price p before service upgrade in Exercise 2.

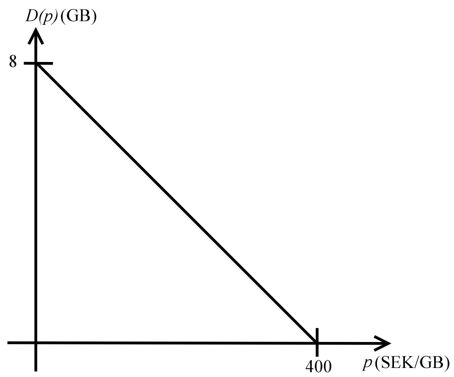

Figure 3: Demand $D(p)$ as a function of price p after service upgrade in Exercise 2.

Exercise 3: An ISP has customers with two different price-demand curves as shown in Fig. 4. The ISP decides that the customers should pay $p = 50$ SEK/GB. The underlying utility function (for both groups of customers) is based on a user poll where the users have told how much they are willing to pay for different amounts of data; that the utility function is concave; that the derivative of the utility function is invertible; and that the utility function evaluated for 0 GB is 0.

a) What is the net utility measured in SEK for both groups of customers under usage-based pricing?
b) What is the net utility measured in SEK for both groups of customers under flat rate?
c) Which type of pricing is beneficial for both customers groups and why?

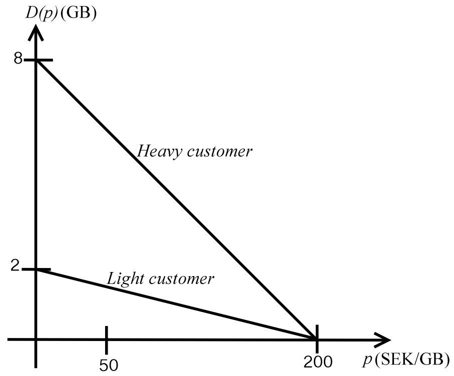

Figure 4: Demand $D(p)$ as a function of price p in Exercise 3.

Exercise 4: An ISP is charging its customers 500 SEK per month (flat rate) based on the peak demand of its heavy user as 50 SEK/GB (shown in Fig.5). There is a light user with a peak demand of 4 GB but is consuming only 3 GB for the current month, while the heavy user is consuming (for the current month) the same peak amount. The underlying utility function (for both customers) is based on a user poll where the users have told how much they are willing to pay for different amounts of data; that the utility function is concave; that the derivative of the utility function is invertible; and that the utility function evaluated for 0 GB is 0.

a) What is the net utility for the current month measured in SEK for heavy user?
b) What is the net utility for the current month measured in SEK for light user?

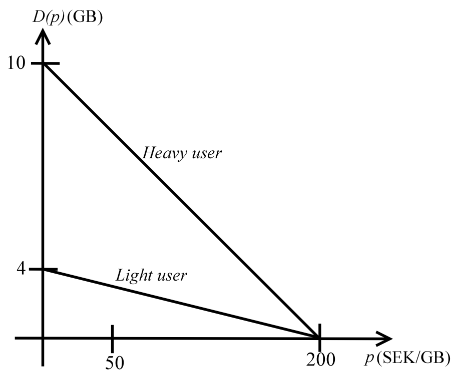

Figure 5: Demand $D(p)$ as a function of price p in Exercise 4.

Exercise 5: An ISP wants to even out the capacity demand over day and night by rewarding its users for delaying their data transmission. Suppose there are two types of users, type A and type B, which have different levels of willingness to delay their data transmission. Their demand during day-time is $d_{A,day}$ = 10 GB for type A users and $d_{B,day}$= 6 GB for type B users.

The demand during night-time is $d_{A,night} = 4$ GB for type A and $d_{B,night} = 3$ GB for type B. Suppose the ISP's capacity is 12 GB during each time period and suppose that the cost of exceeding capacity is 100 SEK per GB. The ISP wishes to give a reward of p SEK per GB for day-time users who delay their data transmission until night-time. Let $q_A(p)$ and $q_B(p)$ be the proportions of data from type A and type B users respectively to delay their transmission given the reward p with:

$$q_A(p) = 1 - \exp(-\frac{p}{3}), \quad q_B(p) = 1 - \exp(-\frac{p}{5}).$$

The ISP wishes to find out the optimal reward price to minimize its total cost, i.e., the sum of cost due to exceeding capacity and the rewards given out.

a) Formulate the minimization problem by writing out the objective function and the constraints.

b) Comment on how to find the optimal rewarding price.