Bell Inequality for Position and Time

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The quantum-mechanical uncertainty in the position of a particle or the time of its emission is shown to produce observable effects that are inconsistent with any local hidden-variable theory. A new experimental test of local hidden-variable theories based on optical interference is proposed.

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Einstein, Podolsky, and Rosen[1] argued that those properties of a system not precisely specified by the quantum theory may have well-defined values determined by some additional variables in a more complete theory. Bell[2] later showed that all local hidden-variable theories are inconsistent with the quantum-theory predictions for the correlations between the spins or polarizations of two distant particles, which have now been verified in a number of experiments. It has been pointed out, however, that violations of Bell's inequality need not be limited[3] to spins or polarizations and that the existing experiments are dependent upon various auxiliary assumptions.[4] As a result, there has been an interest in the possibility of further experimental tests involving other observables and larger spatial separations.

Ghosh and Mandel[5] have recently observed nonlocal effects that correspond to interference between the probability amplitudes for the two ways in which a pair of photons can travel from two slits to two detectors. Unfortunately, a test of Bell's inequality in those experiments would also require the use of polarizers.[6]

This paper derives a new violation of Bell's inequality that is dependent upon interference between the probability amplitudes for a pair of photons to have been emitted at various times by an excited atom. The results show that the quantum-mechanical uncertainty associated with the usual wave-packet description of a particle is inconsistent with any local hidden-variable theory. A new experimental test based on optical interference is proposed, with the interesting feature that the predicted interference occurs for optical path differences much larger than the usual (first-order) coherence length.

The inequality is based on the three-level system shown in Fig. 1. At time $t=0$ an atom is assumed to have been excited into the upper state ψ_1, which has a relatively long lifetime τ_1. After emission of a photon γ_1 with wavelength λ_1, the atom will be in the intermediate state ψ_2, which has a relatively short lifetime $\tau_2 \ll \tau_1$. Thus a second photon γ_2 with wavelength λ_2 will be emitted very soon after γ_1, and a coincidence counting experiment would show a very narrow peak with a width $\sim\tau_2$. The final state ψ_3 is assumed to either have a very long lifetime τ_3 or to be the ground state. In principle, γ_1 and γ_2 could be any massless particles emitted by any quantum system, since the results do not depend on their properties. As a practical matter, atomic transitions frequently do satisfy the condition $\tau_2 \ll \tau_1$.

Photons γ_1 and γ_2 are then collimated by lenses L_1 and L_2 into beams which propagate toward distant detectors D_1 and D_2, respectively, as illustrated in Fig. 2. Spectral filters F_1 and F_2 transmit only wavelengths λ_1 and λ_2. Half-silvered mirrors M_1, M_1', M_2, and M_2' can be inserted into the beams, but the situation without those mirrors will be considered first.

In the absence of the half-silvered mirrors, the coincidence counting rate will simply show a narrow peak indicating that γ_1 and γ_2 were emitted at times which were the same to within a small uncertainty $\sim\tau_2$. The quantum-mechanical description of this process is highly nonlocal, however, since the time at which either photon was emitted was initially uncertain over a much larger time interval $\sim\tau_1$. As a result, the two photons must initially be described by wave packets in which their time of emission and thus their position is relatively uncertain. The detection of one of the photons, say γ_1, immediately determines the time of emission of the other photon and thus its position to within a much smaller uncertainty, which must be reflected by a nonlocal change in the wave function describing the other photon. This nonlocal reduction of the wave function is analogous to that which occurs in the polarization measurements of Bell's origi-

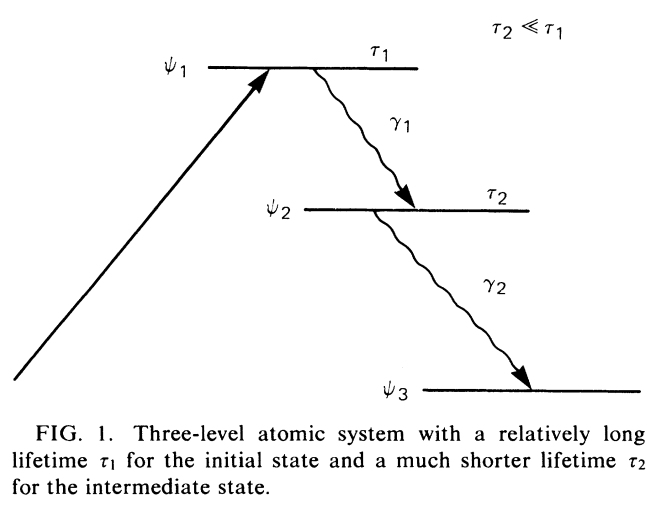

FIG. 1. Three-level atomic system with a relatively long lifetime τ_1 for the initial state and a much shorter lifetime τ_2 for the intermediate state.

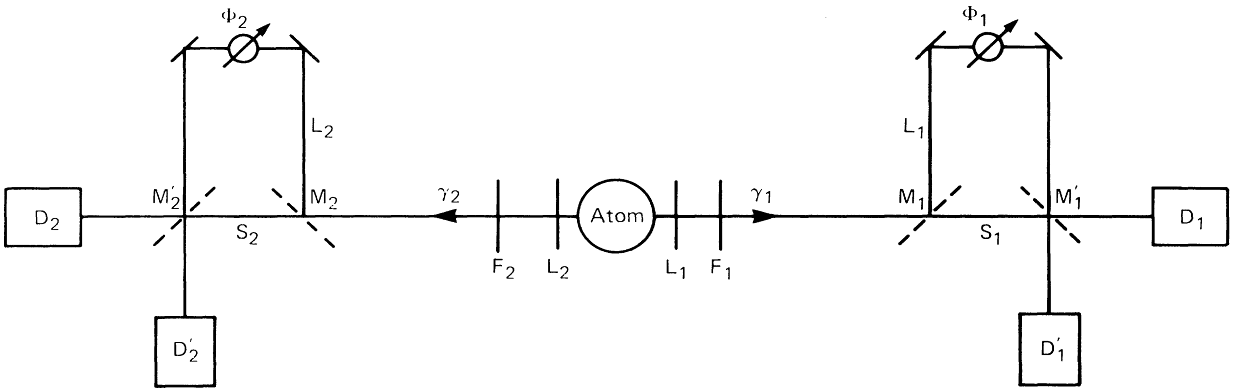

FIG. 2. Photon coincidence measurements including interference between the amplitudes along the shorter paths, S_1 and S_2, and the longer paths, L_1 and L_2.

nal theorem, but here the reduction affects the position and time of emission rather than the polarization.

Although local theories do not allow such instantaneous changes in distant fields, the results could conceivably be consistent with some hidden-variable theory in which the times of emission of the two photons were actually determined all along. The fact[7] that single photons produce interference patterns over distances much larger than $c\tau_1$ suggests, however, that the wavelike nature of the photons cannot be neglected and that hidden-variable theories may be incapable of describing the situation in its entirety.

This will now be shown to be the case by considering the modified coincidence experiment with the half-silvered mirrors in place. M_1 and M_2 split the beams equally into components which travel along either the shorter paths to the detectors, S_1 and S_2, or the longer paths, L_1 and L_2. The difference ΔT in the transit times via the longer and shorter paths is assumed to be the same for both photons and is chosen to satisfy the condition

$$\tau_2 \ll \Delta T \ll \tau_1 \,. \qquad (1)$$

Phase-shift plates Φ_1 and Φ_2 are used to introduce variable phase shifts ϕ_1 and ϕ_2 into the two beams. Half-silvered mirrors M_1' and M_2' recombine the two components, with one set of recombined beams traveling toward D_1 and D_2 as before. For simplicity, the detection efficiencies will be assumed to be 1.0, in which case any particles not detected in D_1 or D_2 will be detected in D_1' or D_2' instead.

A calculation of the coincidence rates predicted by quantum mechanics for this situation is complicated somewhat by the fact that the localization of the particles in space and time requires the use of second-quantized field operators. If the quantum system is assumed to emit massless particles with no spin or polarization, the relevant field operator is

$$\psi(\mathbf{r}) = \sum_{\mathbf{k}} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{V}} a_{\mathbf{k}} \,, \qquad (2)$$

where $a_{\mathbf{k}}^{\dagger}$ creates a particle with momentum $\mathbf{k}$ and V is a large volume containing the system. The derivation which follows will be based on Eq. (2) instead of the electric field operator appropriate for photons in order to explicitly demonstrate the lack of dependence on the polarization and to simplify the notation somewhat. It should be apparent, however, that the same results would be obtained using the electric field operator.

It will be convenient to adopt the Heisenberg representation where the operators evolve in time while the states remain constant, in which case the time-dependent field operator is given by

$$\psi(\mathbf{r}_1,t) = e^{iHt/\hbar}\psi(\mathbf{r})e^{-iHt/\hbar} \,, \qquad (3)$$

where H is the Hamiltonian of the system. At $t=0$ the particle field is in the vacuum state $|0\rangle$, so that the probability amplitude to detect a particle at position $\mathbf{r}$ is subsequently given by $\psi(\mathbf{r},t)|0\rangle$.

Although Eq. (3) can be explicitly solved using perturbation theory, all of the necessary properties of the field can be deduced from well-known and experimentally verified phenomena. For example, once the particles have been emitted, they simply propagate at the speed of light toward the detectors, so that

$$\psi(x + c\Delta t, t) = \psi(x, t-\Delta t) \,, \qquad (4)$$

where x is the distance from the source. In addition, the fact that the coincidence rate is negligibly small for time offsets much larger than τ_2 requires that

$$\psi_0(\mathbf{r}_1,t)\psi_0(\mathbf{r}_2,t\pm\Delta T)|0\rangle = 0 \quad (\Delta T \gg \tau_2) \,. \qquad (5)$$

Here $\mathbf{r}_1$ and $\mathbf{r}_2$ are the locations of the detectors, which are assumed to be equidistant from the source, and $\psi_0(\mathbf{r},t)$ denotes the field operator with the half-silvered mirrors removed.

With the mirrors inserted, the field at detector D_1 is given by

$$\psi(\mathbf{r}_1,t) = \tfrac{1}{2}\psi_0(\mathbf{r}_1,t) + \tfrac{1}{2}e^{i\phi_1}\psi_0(\mathbf{r}_1,t-\Delta T) \,, \qquad (6)$$

where use has been made of Eq. (4) and any phase shifts associated with the half-silvered mirrors have been included in ϕ_1. Similarly,

$$\psi(\mathbf{r}_2,t)=\tfrac{1}{2}\psi_0(\mathbf{r}_2,t)+\tfrac{1}{2}e^{i\phi_2}\psi_0(\mathbf{r}_2,t-\Delta T)\,. \tag{7}$$

The coincidence rate R_c between D_1 and D_2 with the mirrors inserted can now be calculated from

$$R_c=\eta_1\eta_2\langle 0|\psi^\dagger(\mathbf{r}_1,t)\psi^\dagger(\mathbf{r}_2,t)\psi(\mathbf{r}_2,t)\psi(\mathbf{r}_1,t)|0\rangle\,, \tag{8}$$

where η_1 and η_2 are the detection efficiencies of D_1 and D_2. Making use of Eqs. (5)-(7) gives

$$R_c=\tfrac{1}{16}\eta_1\eta_2\langle 0|[\psi_0^\dagger(\mathbf{r}_1,t)\psi_0^\dagger(\mathbf{r}_2,t)+e^{-i\phi_1}e^{-i\phi_2}\psi_0^\dagger(\mathbf{r}_1,t-\Delta T)\psi_0^\dagger(\mathbf{r}_2,t-\Delta T)]$$
$$\times[\psi_0(\mathbf{r}_1,t)\psi_0(\mathbf{r}_2,t)+e^{i\phi_1}e^{i\phi_2}\psi_0(\mathbf{r}_1,t-\Delta T)\psi_0(\mathbf{r}_2,t-\Delta T)]|0\rangle\,. \tag{9}$$

For a time delay $\Delta T\ll\tau_1$, the amplitude to detect a pair of particles at time $t-\Delta T$ will be very nearly equal in magnitude to the amplitude to detect a pair of particles at time t, and the two amplitudes will differ only by a constant phase factor. This can be shown to be the case by writing the relevant part of the field operator at time t in the general form

$$\psi_0(\mathbf{r}_1,t)\psi_0(\mathbf{r}_2,t)=\sum_{k_1}\sum_{k_2}c_{k1,k2}e^{i(\mathbf{k}_1\cdot\mathbf{r}_1-\omega_1 t)}e^{i(\mathbf{k}_2\cdot\mathbf{r}_2-\omega_2 t)}\,, \tag{10}$$

where the coefficients $c_{k1,k2}$ are determined by Eq. (3). Then

$$\psi_0(\mathbf{r}_1,t-\Delta T)\psi_0(\mathbf{r}_2,t-\Delta T)=\sum_{k_1}\sum_{k_2}c_{k1,k2}e^{i(\omega_1+\omega_2)\Delta T}e^{i(\mathbf{k}_1\cdot\mathbf{r}_1-\omega_1 t)}e^{i(\mathbf{k}_2\cdot\mathbf{r}_2-\omega_2 t)}\,. \tag{11}$$

But conservation of energy from the initial to final states requires that

$$\omega_1+\omega_2=(E_1-E_3)/\hbar+\Delta\omega\,, \tag{12}$$

where E_1 and E_3 are the unperturbed energies of states ψ_1 and ψ_3, respectively. The uncertainty $\Delta\omega$ in the sum of the frequencies is due to the finite lifetimes of ψ_1 and ψ_3, and is on the order of

$$\Delta\omega\sim\frac{1}{\tau_1}+\frac{1}{\tau_3}\,. \tag{13}$$

It is important to note that the uncertainty in $\omega_1+\omega_2$ is much less than the individual uncertainty in either ω_1 or ω_2, since the former is unaffected by the relatively short lifetime of the intermediate state ψ_2. For $\Delta T\ll\tau_1$ and $\Delta T\ll\tau_3$, Eq. (13) gives $\Delta\omega\Delta T\ll 1$ and Eq. (11) reduces to

$$\psi_0(\mathbf{r}_1,t-\Delta T)\psi_0(\mathbf{r}_2,t-\Delta T)=e^{i(E_1-E_3)\Delta T/\hbar}\psi_0(\mathbf{r}_1,t)\psi_0(\mathbf{r}_2,t)\,. \tag{14}$$

Equation (14) shows that the relative phase of these two amplitudes is coherent, which gives rise to interference in the coincidence rate despite the fact that the difference in path lengths is larger than the usual (first-order) coherence length. Inserting Eq. (14) into Eq. (9) gives

$$R_c=\tfrac{1}{16}R_0[1+e^{-i(\Delta E\Delta T/\hbar+\phi_1+\phi_2)}][1+e^{i(\Delta E\Delta T/\hbar+\phi_1+\phi_2)}]\,, \tag{15}$$

where R_0 is the coincidence rate with the half-silvered mirrors removed and $\Delta E=E_1-E_3$. Equation (15) can be rewritten as

$$R_c=\tfrac{1}{4}R_0\cos^2\left(\frac{\Delta E\Delta T/\hbar+\phi_1+\phi_2}{2}\right)=\tfrac{1}{4}R_0\cos^2(\phi_1'-\phi_2')\,, \tag{16}$$

where ϕ_1' and ϕ_2' are defined by

$$\phi_1'=\phi_1/2\,,\qquad \phi_2'=-(\phi_2+\Delta E\Delta T/\hbar)/2\,. \tag{17}$$

The quantum-mechanical predictions of Eq. (16) imply that the coincidence counts in the two detectors can be either totally correlated or anticorrelated, depending on the relative settings of the two phase shifters. The form of R_c in Eq. (16) is identical to that obtained in the earlier polarization experiments based upon Bell's theorem, where ϕ_1' and ϕ_2' correspond instead to the orientation of distant polarizers. Bell's proof that no local hidden-variable theory can be consistent with a coincidence rate of this form is independent of the nature of the adjustable parameters ϕ_1' and ϕ_2' associated with the measurement apparatus. The same comments apply to the form of the inequality derived by Clauser, Horne, Shimony, and Holt,[8] which is more suitable for experimental tests. Thus the quantum-mechanical predictions violate Bell's inequality and are inconsistent with any local hidden-variable theory.

An actual experiment would have to take into account the fact that Eq. (1) can only be satisfied approximately, the mechanical stability required to hold ΔT constant, and the effects of collisions and Doppler shift, all of which are beyond the intended scope of this paper. In the author's opinion the experiment is difficult but feasible.

The nonlocal results obtained above are clearly dependent on interference between the probability amplitudes for the pair of particles to have been emitted at two different times and demonstrate that the times of emission cannot be thought of as being well determined by a local hidden-variable theory. Similar comments apply to the positions of the particles. Thus the uncertainties inherent in the usual wave-packet description of a particle have measurable effects that are inconsistent with any local hidden-variable theory.

Finally, it should be noted that one way in which a local hidden-variable theory might fail to agree with the quantum theory is to simply not satisfy Eq. (5) at large distances from the source. In that case, the narrow coincidence peak commonly observed over relatively short distances would "wash out" if observed over much larger distances. It has previously been noted that it may be possible to describe these apparently nonlocal phenomena by a dynamic,[9] local reduction of the wave function, including a possible deterministic interpretation,[10] if phenomena of this kind were limited to relatively small spatial separations. An experimental investigation of these nonlocal phenomena in the limit of large spatial separations would thus be of interest.

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