Testing Born's Rule in Quantum Mechanics with a Triple Slit Experiment ⊘

Urbasi Sinha; Christophe Couteau; Zachari Medendorp; ... et. al

Check for updates

CrossMark

View Online

Export Citation

Articles You May Be Interested In

Born Rule(s)

AIP Conference Proceedings (September 2011)

Stability analysis of numerical solutions of wave propagation when pseudoviscosity is replaced by real viscosity

AIP Conference Proceedings (May 1996)

Playing a quantum game with a qutrit

AIP Conference Proceedings (December 2014)

Testing Born's Rule in Quantum Mechanics with a Triple Slit Experiment

Urbasi Sinha*, Christophe Couteau*, Zachari Medendorp*, Immo Söllner*,†,
Raymond Laflamme*,**, Rafael Sorkin‡,** and Gregor Weihs*,†

*Institute for Quantum Computing, University of Waterloo, 200 University Ave W,
Waterloo, Ontario N2L 3G1, Canada
†Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25, 6020 Innsbruck, Austria
**Perimeter Institute for Theoretical Physics, 31 Caroline St, Waterloo, Ontario N2L 2Y5, Canada
‡Department of Physics, Syracuse University, Syracuse, NY 13244-1130

Abstract. In Mod. Phys. Lett. A **9**, 3119 (1994), one of us (R.D.S) investigated a formulation of quantum mechanics as a generalized measure theory. Quantum mechanics computes probabilities from the absolute squares of complex amplitudes, and the resulting interference violates the (Kolmogorov) sum rule expressing the additivity of probabilities of mutually exclusive events. However, there is a higher order sum rule that quantum mechanics does obey, involving the probabilities of three mutually exclusive possibilities. We could imagine a yet more general theory by assuming that it violates the next higher sum rule. In this paper, we report results from an ongoing experiment which sets out to test the validity of this second sum rule by measuring the interference patterns produced by three slits and all the possible combinations of those slits being open or closed. We use an attenuated laser light combined with single photon counting to confirm the particle character of the measured light.

Keywords: Probability, Quantum mechanics, Born's rule, Interference
PACS: 03.65.Ta, 42.50.Ct

1. INTRODUCTION AND MOTIVATION

Quantum Mechanics has been one of the most successful tools in the history of Physics. It has revolutionized Modern Physics and helped explain many phenomena. However, in spite of all its successes, experts still believe that there may be some gaps in our understanding of the subject and there may be more to it than meets the eye. This makes it very important to have experimental verifications of all the fundamental postulates of Quantum Mechanics. In this paper, we aim to test Born's interpretation of probability [1], which states that if a quantum mechanical state is specified by the wavefunction $\psi(r,t)$ [2], then the probability that a particle lies in the volume element d^3r located at r and at time t, is given by:

$$\psi^*(r,t)\psi(r,t)d^3r = |\psi(r,t)|^2 d^3r \tag{1}$$

Although this definition of probability has been assumed to be true in describing several experimental results, no experiment has ever been performed to specifically test this definition alone. Even in his Nobel lecture in 1954, Born raised the issue of proving his postulate, but fifty four years have passed without there being a dedicated attempt at such an experimental verification. In this paper, we report results on such an ongoing experiment.

2. THE 3-SLIT EXPERIMENT

In Ref. [3], one of us (R.D.S) proposed the triple slit experiment motivated by the "sum over histories" approach to Quantum Mechanics. According to this approach, Quantum theory differs from classical mechanics in its dynamics, which is stochastic rather than deterministic. There are probabilities associated with sets of histories. Each set A of histories is associated with a non-negative real number $p_A = |A|$ which is called the "quantum measure" (and is the same as the probability). It is this measure (or the probability) that enters the sum rules we are concerned with. Details of the quantum measure theory following a sum over histories approach can be found in [3, 4].

Interference describes the deviation from the classical additivity of mutually exclusive events. A sum rule says that the interference term $I = 0$ which means that if additivity holds, we call it a sum rule. Let us define a hierarchy of interference terms:

$$I_A = p_A \tag{2}$$

$$I_{AB} = p_{AB} - p_A - p_B \tag{3}$$

$$I_{ABC} = p_{ABC} - p_{AB} - p_{BC} - p_{CA} + p_A + p_B + p_C \tag{4}$$

Equations (2), (3), and (4) refer to the zeroth, first, and second sum rule respectively. Here, p_{ABC} means the probability of the disjoint union of the sets A, B, and C. A physical system in which such probability terms appear would be a system with three paths [5], for example three slits A, B and C in an opaque aperture. For particles incident on the slits, p_A would refer to the probability of a particle being detected at a chosen detector position having travelled through slit A and p_B and p_C would refer to similar probabilities through slits B and C.

The zeroth sum rule needs to be violated ($I_A \neq 0$) for a non-trivial measure. If the first sum rule holds, i.e. $I_{A,B} = 0$, it leads to regular probability theory for example for classical stochastic processes. Violation of the first sum rule ($I_{A,B} \neq 0$) is consistent with Quantum Mechanics. A sum rule always entails that the higher ones in the hierarchy hold. However, since the first sum rule is violated in Quantum Mechanical systems, one needs to go on to check the second sum rule. In known systems, triadditivity of mutually exclusive probabilities is true i.e., the second sum rule holds, $I_{A,B,C} = 0$. This follows from algebra as shown below and is based on the assumption that Born's rule holds.

$$
\begin{aligned}
p_{ABC} &= |\psi_A + \psi_B + \psi_C|^2 \\
&= |\psi_A|^2 + |\psi_B|^2 + |\psi_C|^2 + \psi_A^*\psi_B + \psi_B^*\psi_A + \psi_B^*\psi_C + \psi_C^*\psi_B + \psi_A^*\psi_C + \psi_C^*\psi_A \\
&= p_A + p_B + p_C + I_{AB} + I_{BC} + I_{CA} \\
&= p_A + p_B + p_C + (p_{AB} - p_A - p_B) + (p_{BC} - p_B - p_C) + (p_{CA} - p_C - p_A) \\
&= p_{AB} + p_{BC} + p_{CA} - p_A - p_B - p_C \tag{5} \\
\Rightarrow I_{ABC} &\equiv p_{ABC} - p_{AB} - p_{BC} - p_{CA} + p_A + p_B + p_C = 0 \tag{6}
\end{aligned}
$$

If however, there is a higher order correction to Born's rule (however small that correction might be), equation (5) will lead to a violation of the second sum rule. The triple slit experiment proposes to test the second sum rule and for this purposed we define a quantity ε as

$$\varepsilon = p_{ABC} - p_{AB} - p_{BC} - p_{CA} + p_A + p_B + p_C - p_0. \tag{7}$$

Figure 1 shows how the various probabilities could be measured in a triple slit configuration. As opposed to the ideal formulation where empty sets have zero measure, we need to provision for a non-zero p_0, the background probability of detecting particles when all paths are closed. This takes care of any experimental background, such as detector dark counts. For better comparison between possible realizations of such an experiment, we further define a normalized variant of ε called ρ,

$$\rho = \frac{\varepsilon}{\delta}, \text{ where} \tag{8}$$
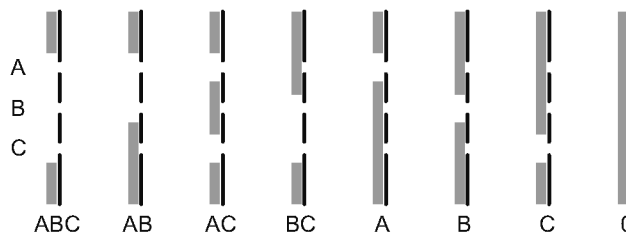
FIGURE 1. Pictorial representation of how the different probability terms are measured

$$\delta = |I_{AB}| + |I_{BC}| + |I_{CA}|$$
$$= |p_{AB} - p_A - p_B + p_0| + |p_{BC} - p_B - p_C + p_0| + |p_{CA} - p_C - p_A + p_0|. \tag{9}$$

Since δ is a measure of the regular interference contrast, ρ can be seen as the ratio of the violation of the second sum rule versus the expected violation of the first sum rule. In the following sections we will describe how we implemented the measurements of all the terms that compose ρ and and analyze our results.
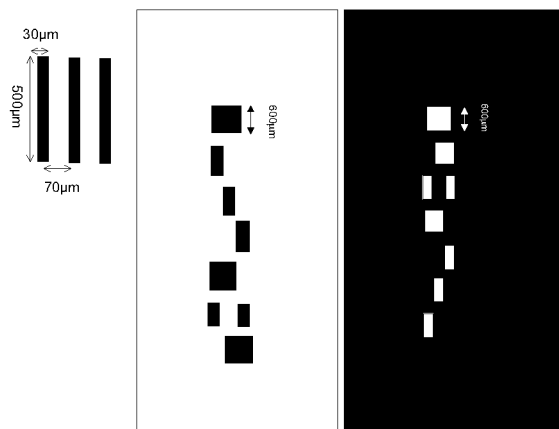
2.1. Making the slits

Our first step in designing the experiment was to find a way to reliably block and unblock the slits, which we expected to be very close together, so that simple shutters wouldn't work. Therefore we decided to use a set of two plates, one containing the slit pattern and one containing patters to block or unblock the slits. The slits were fabricated by etching them on some material which covered a glass plate. The portion of the material which had the slits etched in would be transparent to light and the rest of the glass plate which was still covered would be opaque. However, not all materials exhibit the same degree of opacity to infra-red light and this leads to spurious transmission through portions of the glass plate which should be opaque in theory.

Various types of materials were used for etching the slits and each modification led to a decrease in spurious transmission through the glass plate. At first, a photo-emulsion plate was used which had a spurious transmission of around 5%. This was followed by a glass plate with a chromium layer on top. This had a spurious transmission of around 3%. The plate currently in use has an aluminium layer of 500 nm thickness on top. Aluminium is known to have a very high absorption coefficient for infrared light and this led to a spurious transmission of less than 0.1%.

The blocking patterns were etched on a different glass plate covered with the same material as the first glass plate. Figure 2 shows an example of a set of blocking patterns which would give rise to the eight intensities corresponding to the probability terms related to the 3-slit open, 2-slit open and 1-slit open configurations as discussed in the previous section.

Another way of achieving the eight intensities would be to open up the right number and position of slits instead of blocking them off. This is also shown in Figure 2. This leads to a big change in the appearance of the second glass plate. In the first instance, when the slits were being blocked for the different cases, the rest of the glass plate was transparent and only the portions which were being used for blocking off the slits in the first glass plate were opaque to light. This led to spurious effects as a lot of light was being let through the glass plate this way and this led to appearance of background features in the diffraction patterns. However, with the second design, the whole plate was covered with the opaque material and only portions which were being used to open up slits in the first plate allowed light to go through, thus leading to diminishing background effects.

FIGURE 2. Different ways of measuring the eight intensities. The LHS shows a schematic of a 3 slit pattern. In the center, the first blocking scheme is demonstrated, in which the slits are blocked according to the terms being measured. The whole glass plate is thus transparent with only the blocking portions opaque. The RHS shows the second blocking scheme in which the slits are opened up as needed on a glass plate which is completely opaque except for the unblocking openings.

202
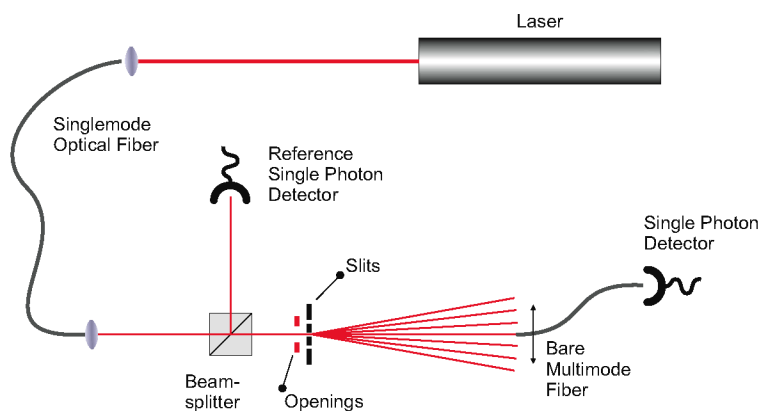
FIGURE 3. Schematic of experimental set-up

2.2. The experimental set-up

Figure 3 shows a schematic of the complete experimental set-up. The He-Ne laser beam passes through an arrangement of mirrors and collimators before being incident on a 50/50 beam splitter. In the near future we will replace the laser by a heralded single photon source [6]. The beam then splits into two, one of the beams is used as a reference arm for measuring fluctuations in laser power whereas the other beam is incident on the glass plate, which has the slit pattern etched on it. The beam height and waist is adjusted so that it is incident on a set of three slits, the slits being centered on the beam. There is another glass plate in front which has the corresponding blocking designs on it such that one can measure the seven probabilities in equation (4). The slit plate remains stationary whereas the blocking plate is moved up and down in front of the slits to yield the various combinations of opened slits needed to measure the seven probabilities. As mentioned above, in our experimental set-up, we also measure an eighth probability which corresponds to all three slits being closed in order to account for dark counts and any background light. Figure **??** shows this pictorially. There is a horizontal microscope (not shown in Figure 3) for initial alignment between the slits and the corresponding openings. A multi-mode optical fiber is placed at a point in the diffraction pattern and connected to an avalanche photo-diode (APD) which measures the photon counts corresponding to the various probabilities. Using an APD ensures particle nature at the detection level. The optical fibre can be moved to different positions in the diffraction pattern in order to obtain the value of ρ at different positions in the pattern. Figure 4 shows a measurement of the eight diffraction patterns corresponding to the eight configurations of open and closed slits as required by equation (7).

2.3. Results

In a null experiment like ours, where we try to prove the existence or absence of an effect, proper analysis of possible sources of errors is of utmost importance. It is essential to have a good estimation of both random errors in experimental quantities as well as possible potential sources of systematic errors. For each error mechanism we calculate within the framework of some accepted theory how much of a deviation from the ideally expected value they will cause. Drifts in time or with repetition can often be corrected by better stabilization of the apparatus, but any errors that do not change in time can only be characterized by additional measurements.

We have measured ρ for various detector points using a He-Ne laser. Initially, the value of ρ showed strong variations with time and this was solved by having better temperature control in the lab and also by enclosing the set-up in a black box so that it is not affected by stray photons in the lab. Fig. 5 shows a recent overnight run which involved measuring ρ around hundred times at a position near the center of the diffraction pattern. Only a slight drift in the mean can be discerned. The typical value of ρ is in the range of $10^{-2} \pm 10^{-3}$. The random error is one order of magnitude lower than the reported mean ρ value and does not explain the deviation of the mean ρ from zero. Next we analyze some systematic errors which may affect our experiment to see if these can be big enough to explain the deviation of ρ from the zero expected from Born's rule.
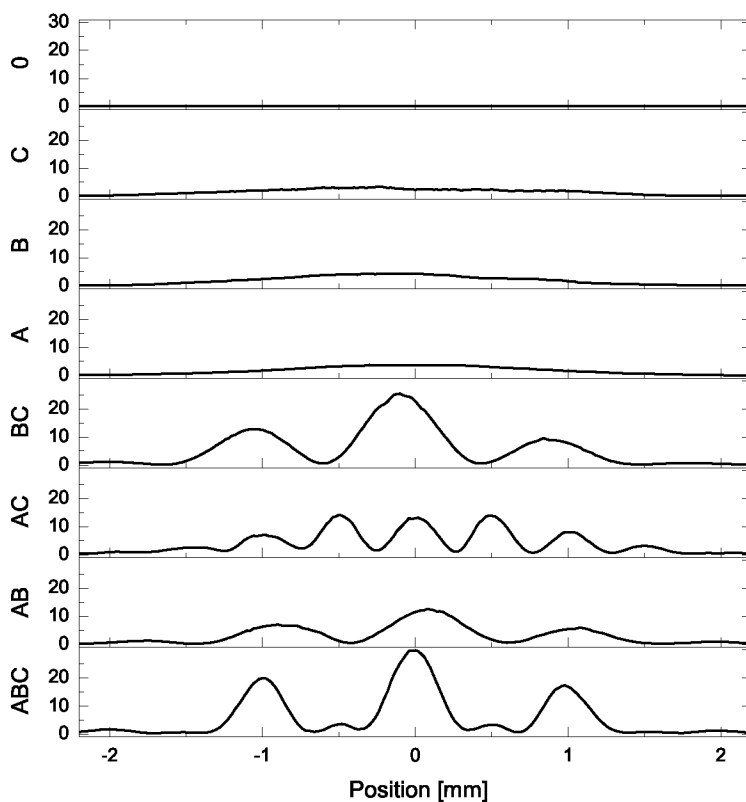
FIGURE 4. Diffraction patterns measured by using a He-Ne laser. The vertical axis is in units of 1000 photocounts.
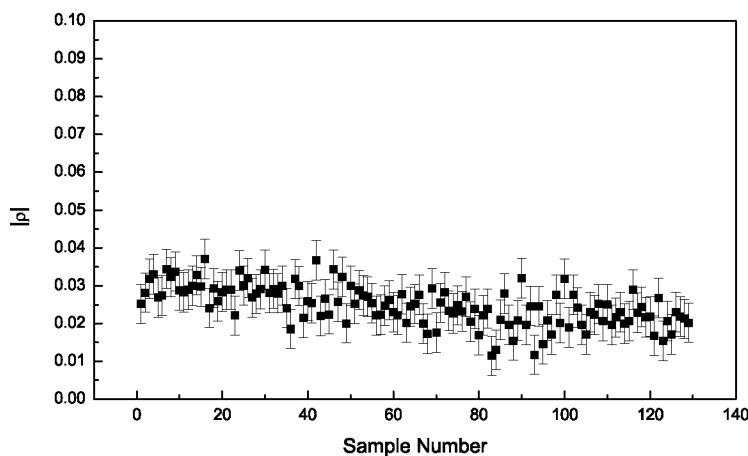

FIGURE 5. Overnight measurement of ρ. Each data point corresponds to approximately 5 min of total measurement time. A slight drift of the mean is visible. The error bars are the size of the standard deviation of the ρ values.

3. ANALYSIS OF SOME POSSIBLE SOURCES OF SYSTEMATIC ERRORS

By virtue of the definition of the measured quantity ε (or its normalized variant ρ) some potential sources of errors do not play a role. For example, it is unimportant whether the three slits in the aperture have the same size, shape, open transmission, or closed transmission. However, in the current set-up we are measuring the eight different combinations of open and closed slits using a blocking mechanism that does not block individual slits but by changing
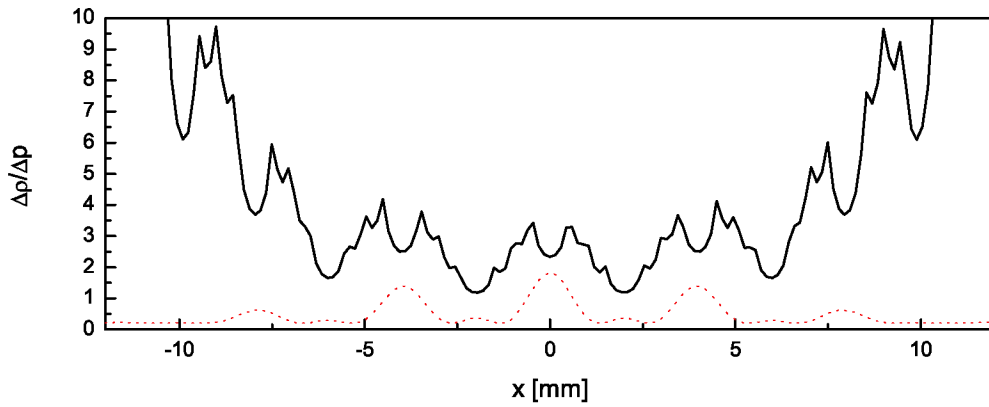
FIGURE 6. Fluctuation $\Delta\rho$ of ρ caused by fluctuating source power (solid line). The horizontal axis is the spatial coordinate in the far field of the three slits. The dotted line shows a scaled three-slit diffraction pattern as a position reference.

a global unblocking mask. Also, the measurements of the different combinations occur sequentially, which makes the experiment prone to the effects of fluctuations and drifts. In the following we will analyze the effects of three systematic error mechanisms, power drifts or uneven mask transmission, spurious mask transmission combined with misalignment, and detector nonlinearities.

The power of light source is never perfectly stable and the fact that we measure the eight individual combinations at different times leads to a difference in the effective energy received by a certain aperture combination over the time interval it is being measured for. Since in practice we don't know how the power will change, and because we may choose a random order of our measurements we can effectively convert this systematic drift into a random error. Conversely, if in the experiment we found that the power was indeed drifting slowly in one direction, then randomization of the measurement sequence would mitigate a non-zero mean.

Let us therefore assume a stationary mean power P and a constant level of fluctuations ΔP around that power for an averaging time that is equal to the time we take to measure one of the eight combinations. Let the relative fluctuation $\Delta p = \Delta P/P$. Using Gaussian error propagation, the fluctuation $\Delta\rho$ of ρ, whose quantum theoretical mean is zero, is then given by

$$(\Delta\rho)^2 = \frac{1}{\delta^2}\left[P_{ABC}^2 + (1+s_{BC}\rho)P_{BC}^2 + (1+s_{AC}\rho)P_{AC}^2 + (1+s_{AB}\rho)P_{AB}^2 + \right. \tag{10}$$
$$(1+(s_{BC}+s_{AC})\rho)P_C^2 + (1+(s_{BC}+s_{AB})\rho)P_B^2 + (1+(s_{AC}+s_{AB})\rho)P_A^2 +$$
$$\left.(1+(s_{BC}+s_{AC}+s_{AB})\rho)P_0^2\right](\Delta p)^2,$$

where the quantities s are the signs of the binary interference terms that appear in δ, e.g. $s_{AB} = \text{sign}(I(A,B))$. Fig. 6 shows a plot of $\Delta\rho/\Delta p$ as a function of the position in the diffraction pattern. The curve has divergences wherever δ has a zero. These are the only points that have to be avoided. Otherwise, the relative power stability of the source translates with factors close to unity into the relative error of ρ.

Obviously, Eq. 11 is also exactly the formula for the propagation of independent random errors of any origin in the measurements, if they are all of the same relative magnitude. However, if we use a photon counting technique, then the random error of each measurement follows from the Poissonian distribution of the photocounts. In this case, the (relative) random error of P_x is proportional to $1/\sqrt{P_x}$, where x is any of the eight combinations. As a consequence random error of ρ will be proportional to the same expression with all the P_x^2 replaced by P_x.

While it appears that drifting or fluctuating power can be mitigated, a worse problem is that in our realization of the unblocking of slits every pattern could potentially have slightly different transmission. Possible reasons for this are dirt, or incomplete etching of the metal layer, or inhomogeneities in the glass substrate or the antireflection coating layers. In order to avoid any of these detrimental possibilities the next implementation of the slits will be air slits in a steel membrane.

As a second source of systematic errors we have identified the unwanted transmission of supposedly opaque parts of the slit and blocking mask. This by itself would not cause a non-zero ρ, but combined with small errors in the alignment of the blocking mask it will yield aperture transmission functions that are not simply always the same open

205
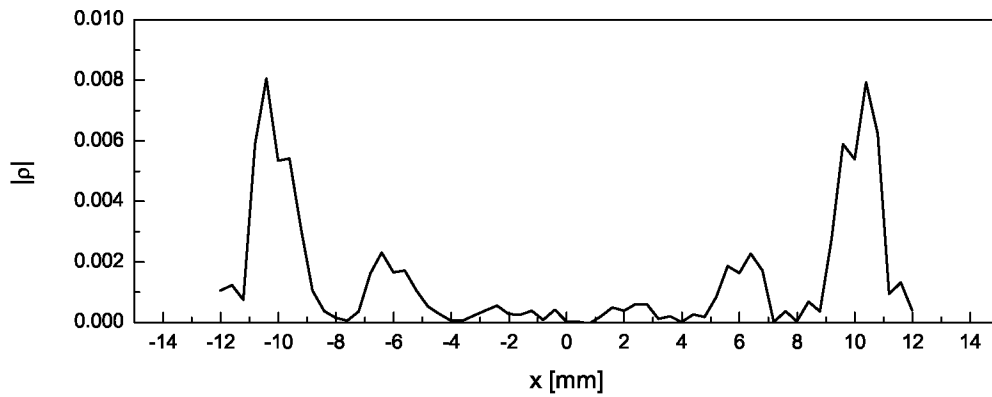
FIGURE 7. Value of ρ in the diffraction pattern of three slits for the following set of parameters: 30 μm slit size, 100 μm slit separation, 800 nm wavelength, 100 μm opening size, 5% unwanted mask transmission, and a set of displacements of the blocking mask uniformly chosen at random from the interval [0,10 μm]
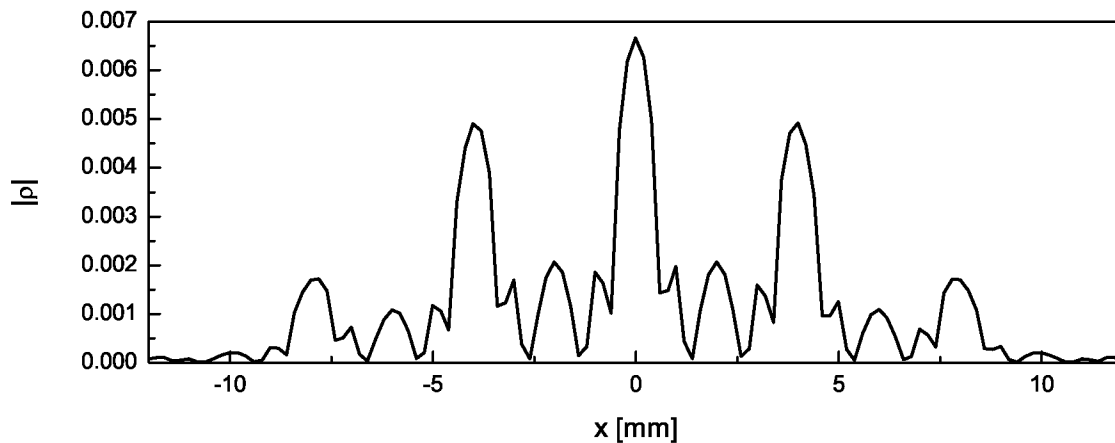
FIGURE 8. Value of ρ in the diffraction pattern of three slits for a 0.5% nonlinear detector over, where the ratio between the maximum detected power and the minimum detector power is 100.

and closed slit, but in every one of the eight combinations we have a particular aperture transmission function. If the slits were openings in a perfectly opaque mask there would be no effect, since they are not being moved between the measurements of different combinations. In practice, we found that all our earlier masks had a few percent of unwanted transmission as opposed to the current one which has an unwanted transmission smaller than 0.1%. Fig. 7 shows the results of a simulation assuming the parameters of the current mask, which seems to be good enough to avoid this kind of systematic error at the current level of precision.

Finally, there is a source of systematic error, which is intrinsically linked to the actual objective of this measurement. We set out to check the validity of Born's rule, that probabilities are given by absolute squares of amplitudes. Yet, any real detector will have some nonlinearity. In a counting measurement the effect of dead-time will limit the linearity severely, even at relatively low average count rates. A typical specification for an optical power meter is 0.5% nonlinearity within a given measurement range. The measurement of all eight combinations involves a large dynamic range. From the background intensity to the maximum with all three slits open, this could be as much as six orders of magnitude. Fig. 8 shows that 1% nonlinearity translates into a non-zero value of ρ of up to 0.007. For the measurements shown above the mean count rate was about 80,000 counts per second. Given a specified dead time of our detector of 50 ns, we expect the deviation from linearity to be about 0.4% and the resulting apparent value of $\rho \approx 0.003$.

All of these systematics are potential contributors to a non-zero mean ρ. From the above calculations and our efforts to stabilize the incident power and improvements in the mask properties, we conclude that while detector nonlinearities

may have contributed some the main source of systematic error must the inhomogeneities in the unblocking mask. Hopefully air slits will bring a significant improvement.

4. DISCUSSION, CONCLUSION AND FUTURE WORK

In this experiment, we have attempted to test Born's rule for probabilities. This is a null experiment but due to experimental inaccuracies, we have measured a value of ρ which is in the $10^{-2} \pm 10^{-3}$ range. We have analyzed some major sources of systematic errors that could affect our experiments and we will try to reduce their influence in future implementations. Further, we plan replace the random laser source by a heralded single photon source [6]. This will ensure the particle nature of light both during emission and detection and give us the advantage that we can count the exact number of particles entering the experiment. At this point we don't know of any other experiment that has tried to test Born's rule using three-path interference, therefore we cannot judge how well we are doing. However, our collaborators [7] are undertaking an interferometric experiment using neutrons, which will perform the test in a completely different system. These two approaches are complementary and help us in our quest to estimate the extent of the validity of the Born interpretation of the wavefunction.

5. ACKNOWLEDGEMENTS

REFERENCES

1. M. Born, *Zeitschrift für Physik* **37**, 863–867 (1926).
2. E. Schrödinger, *Phys. Rev.* **28**, 1049–1070 (1926).
3. R. D. Sorkin, *Modern Physics Letters A* **9**, 3119–3127 (1994), `arXiv:gr-qc/9401003`.
4. R. D. Sorkin, "Quantum Measure Theory and its Interpretation," in *Quantum Classical Correspondence: Proceedings of the 4th Drexel Symposium on Quantum Nonintegrability*, edited by D. Feng, and B.-L. Hu, International Press, Cambridge, MA, 1997, pp. 229–251.
5. G. Weihs, M. Reck, H. Weinfurter, and A. Zeilinger, *Optics Lett.* **21**, 302–304 (1996).
6. E. Bocquillon, C. Couteau, M. Razavi, R. Laflamme, and G. Weihs, Coherence measures for heralded single-photon sources (2008), `arXiv:0807.1725`.
7. D. G. Cory (2008), private communication.