Kit nr 3: The Deutsch-Jozsa algorithm

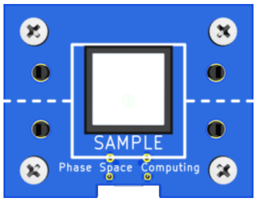
Student Manual

2021-04-28

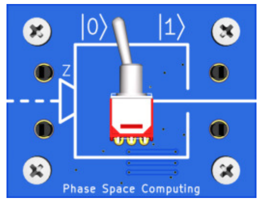
This lab kit contains material to demonstrate the Deutsch-Jozsa quantum algorithm up to three qubit inputs.

You should read through the manual before starting the laboratory work.

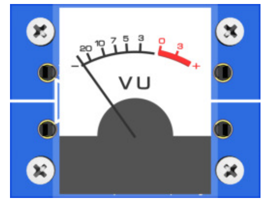
1 Content of the kit



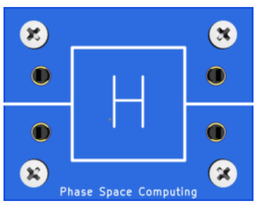
Sample button $\times 1$**:** Resamples the random numbers used in the simulation.



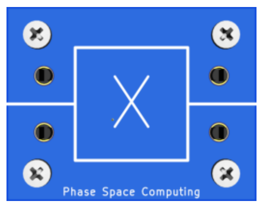
Source $\times 4$**:** component that sends a qubit in one of the computational states $|0\rangle$ or $|1\rangle$ depending on the setting of the switch.



Measurement device $\times 3$**:** Measures the quantum state and displays the result. The reading is $|0\rangle$ if the meter points to the left, and $|1\rangle$ if the meter points to the right (into the red area).

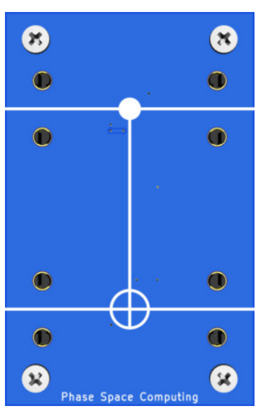


Hadamard gate $\times 7$**:** Transforms states in the $|0\rangle/|1\rangle$ basis to the $|+\rangle/|-\rangle$ basis and vice versa.

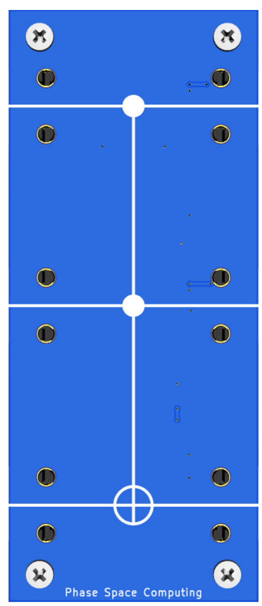


Pauli-X gate $\times 1$**:** Inverts the states of the computational basis, $|0\rangle \leftrightarrow |1\rangle$.

Power cord $\times 1$
Patch cable (7 cm) $\times 10$
Patch cable (30 cm) $\times 5$
Patch cable (150 cm) $\times 2$



CNOT gate (or Controlled NOT gate) $\times 3$**:** the gate operates on two qubits. The second qubit (that goes in the lower part of the component) is called target qubit and is flipped by the gate if and only if the first qubit (the control qubit) is $|1\rangle$.



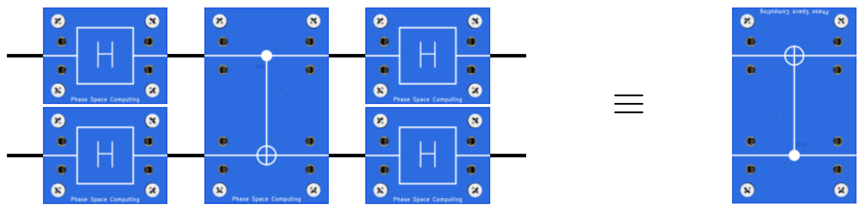
Toffoli $\times 2$**:** the gate operates on three qubits. The third qubit (that goes in the lowest part of the component) is called target qubit and is flipped by the gate if and only if the first two qubits (the control qubits) are $|1\rangle$.

2 Required reading on the Deutsch-Jozsa Quantum Algorithm

Read Sections 1.4, and 4.1–4.4 of M. A. Nielsen and I. L. Chuang (2010). *Quantum Computation and Quantum Information*. Vol. 10th Anniversary Edition. Cambridge University Press. ISBN: 978-1-107-00217-3.

3 Preparatory excercises

1. Verify (theoretically) that the following identity holds,



2. Draw the gate array for the four different functions for the Deutsch problem (balanced or constant functions from one bit to one bit).

3. Draw the gate arrays for the Deutsch quantum algorithm for the arrays of Exercise 2, including the input states. What does Exercise 1 tell you will happen for the balanced functions?

4. How many balanced functions are there for two-bit inputs? (How many constant functions are there?) Use the binary notation $x = x_1x_0$ to resolve the bit values x_1 and x_0, and construct all balanced functions for two-bit inputs using only CNOTs (at most three of them) and one inverter (X). What does Exercise 1 tell you will happen in the Deutsch-Jozsa quantum algorithm for the balanced functions?

5. How many balanced functions are there for three-bit inputs? (How many constant functions are there?) Construct the alternating balanced function $f(x_2x_1x_0) = x_0$. What does Exercise 1 tell you will happen in the Deutsch-Jozsa quantum algorithm for this balanced function?

6. Construct the three-qubit-balanced function that has $f(111) = 0$, $f(110) = 1$, $f(101) = 1$, $f(100) = 1$, $f(011) = 1$, $f(010) = 0$, $f(001) = 0$, and $f(000) = 0$. What does Exercise 1 tell you will happen in the Deutsch-Jozsa quantum algorithm for this balanced function?

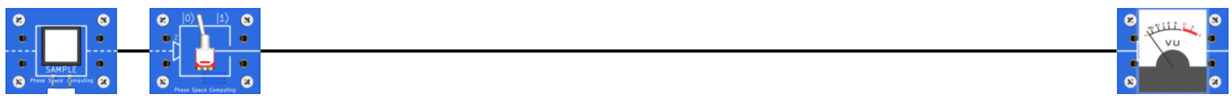
*7. Draw a random balanced function for three-bit inputs. Construct this function using only Toffolis (at most two of them), CNOTs (at most three of them) and one inverter (X) (Tedious, but possible: construct all balanced functions for three-bit inputs in this manner).

4 Laboration tasks

From here, the laboratory work begins. A standard session should include running the Deutsch-Jozsa quantum algorithm for at least one three-bit input balanced function.

Task 1 (Quantum bits): Verify that source and measurement works

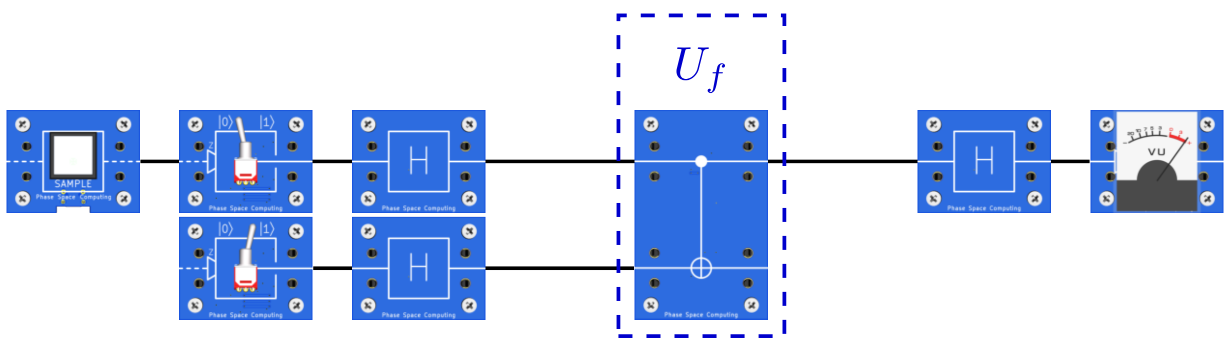
Materials: Power unit, sample button, one source, one detector, one short and one long cable



Connect the source to the detector using the long connector cable, connect the sample button, connect power, and test the circuit by flipping the switch and pressing the send button. When sending a 0, the measurement device should show a low value (the meter should point to the left), and when sending a 1, the measurement device should show a high value (the meter should point to the right, into the red area)

Task 2 Run the Deutsch quantum algorithm for all one-bit input functions

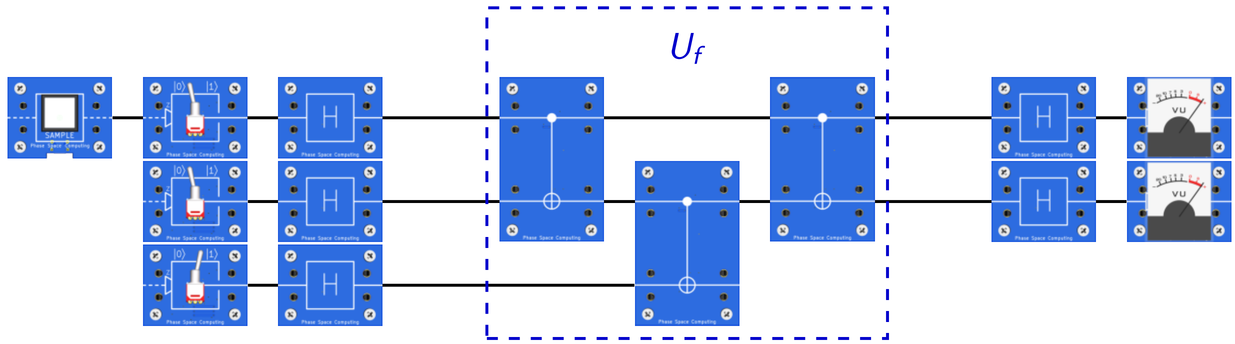
Materials: Power unit, sample button, two sources, two detectors, three Hadamards, one CNOT, cables



Example of the Deutsch quantum algorithm for a single-bit input balanced function. Connect as in the above picture, and verify that the Deutsch algorithm gives the correct measurement outcome. Repeat for all the functions in Excercise 2

Task 3 Run the Deutsch-Jozsa quantum algorithm for two-bit input balanced functions

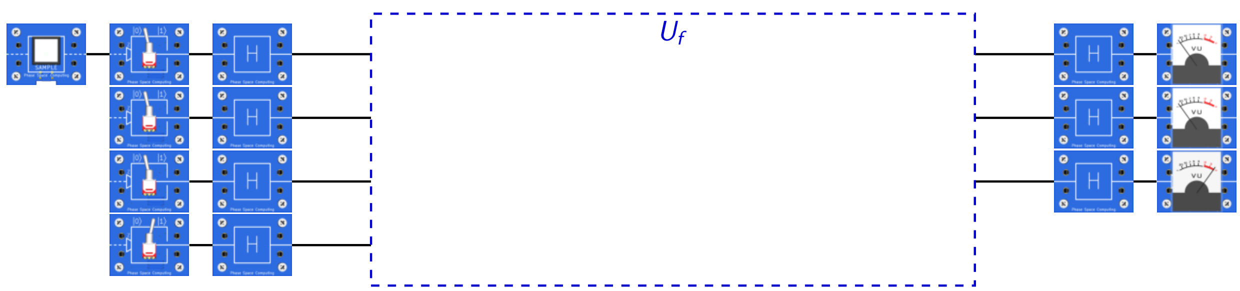
Materials: Power unit, sample button, two sources, two detectors, three Hadamards, one CNOT, cables



Example of the Deutsch-Jozsa quantum algorithm for a two-bit input balanced function. Connect as in the above picture, and verify that the Deutsch-Jozsa quantum algorithm gives the correct measurement outcome. Repeat for all the functions of Excercise 4

Task 4 Run the Deutsch-Jozsa quantum algorithm for some three-bit input balanced functions

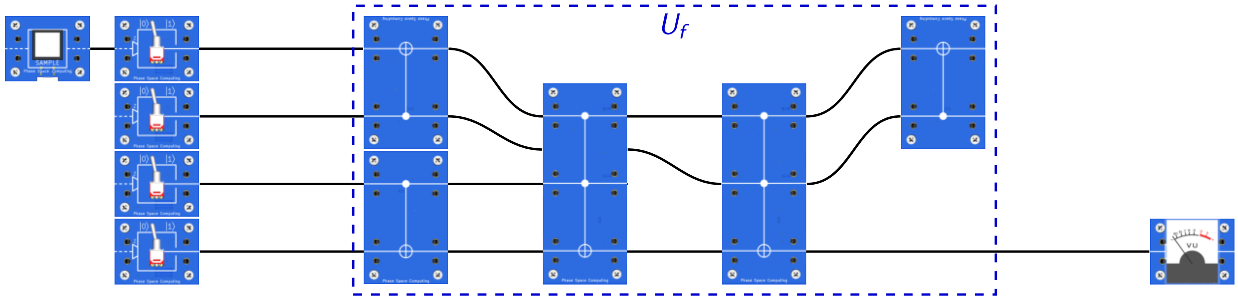
Materials: Power unit, sample button, four sources, three detectors, seven Hadamards, some CNOTs, some Toffolis, cables



Connect to make the alternating balanced function of Exercise 5, and verify that the Deutsch-Jozsa quantum algorithm gives the correct measurement outcome. Repeat for the function of Exercise 6.

Task *5 Construct another three-bit input balanced function, and verify that it is balanced through exhaustive search

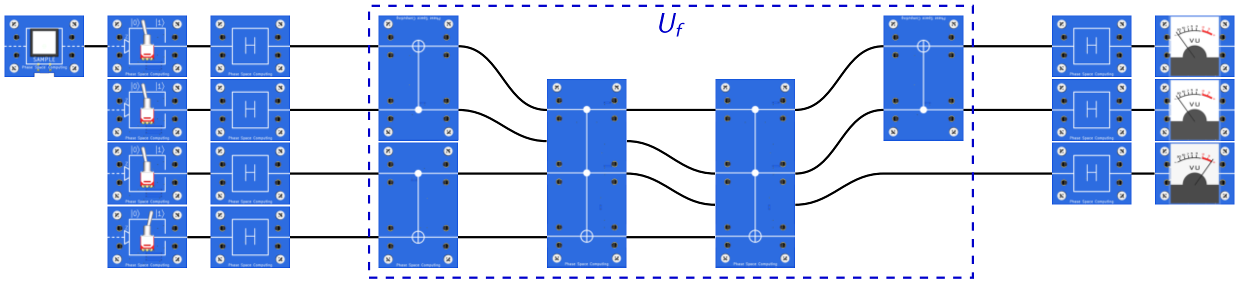
Materials: Power unit, sample button, four sources, one detector, three CNOTs, two Toffolis, cables



Connect as in the above picture, and run through all possible input values to verify that this is a balanced function. What are the outputs?

Task *6 Run the Deutsch-Jozsa quantum algorithm for the function from Task *5

Materials: Power unit, sample button, four sources, three detectors, seven Hadamards, three CNOTs, two Toffolis, cables



Connect Hadamards and detectors to the function of Task *5, and verify that the Deutsch-Jozsa quantum algorithm gives the correct measurement outcome. Is the outcome deterministic?